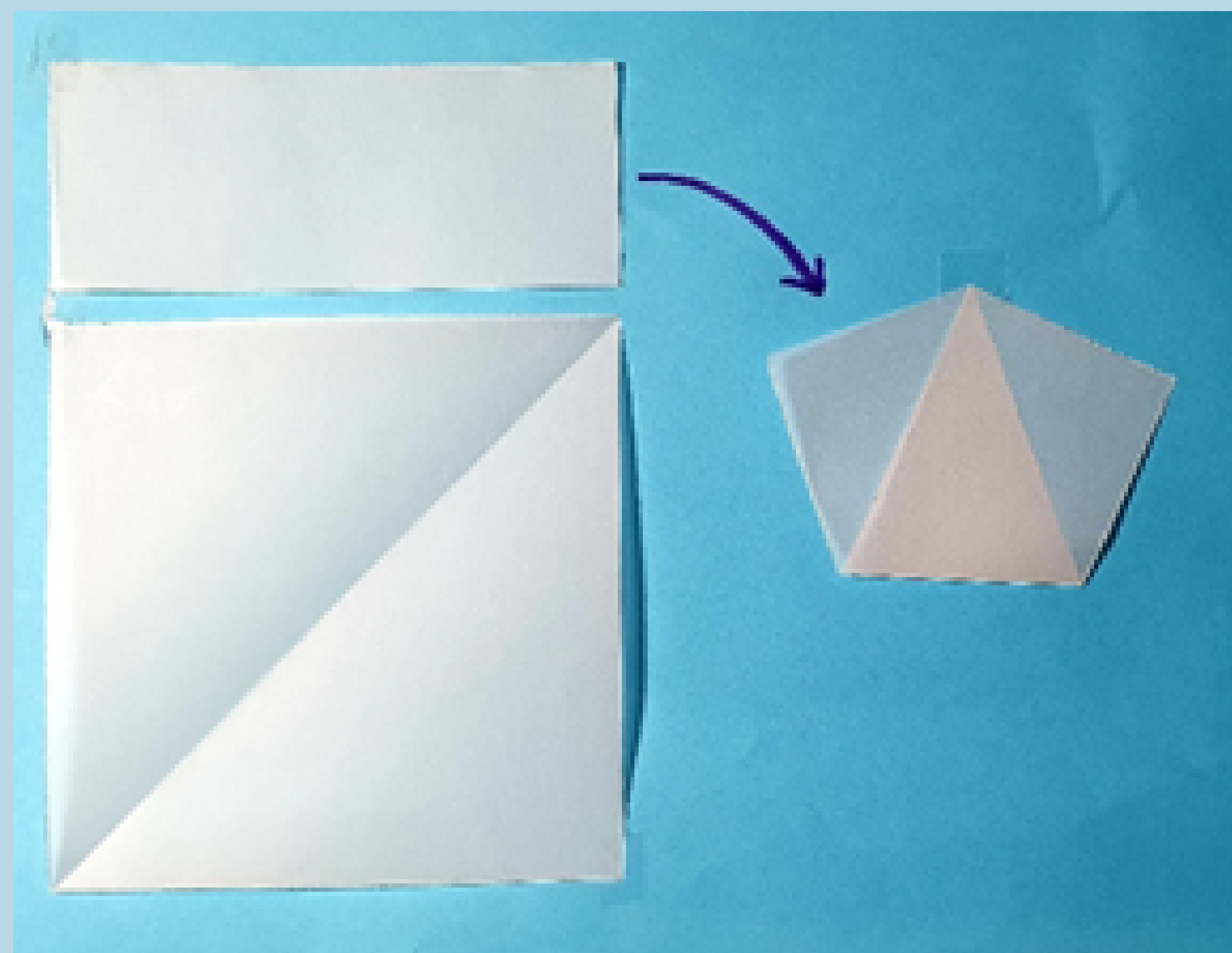


The need for proof is not obvious

Eszter Kónya (University of Debrecen, Hungary)
Zoltán Kovács (Eszterházy Károly Catholic University, Hungary)

Activity

Starting point: a sheet of A4 paper, ratio of sides is $1 : \sqrt{2}$.



Task description

Discover as many properties of the folded pentagon as possible.

Didactic contract

The drawing and the justification of the result are part of the didactic contract.

Method

Participants: 48 prospective teachers

Assignment: Written homework

Data collection: autumn 2020 and spring 2021

The most frequent answers

This pentagon is symmetric and has four equal sides.

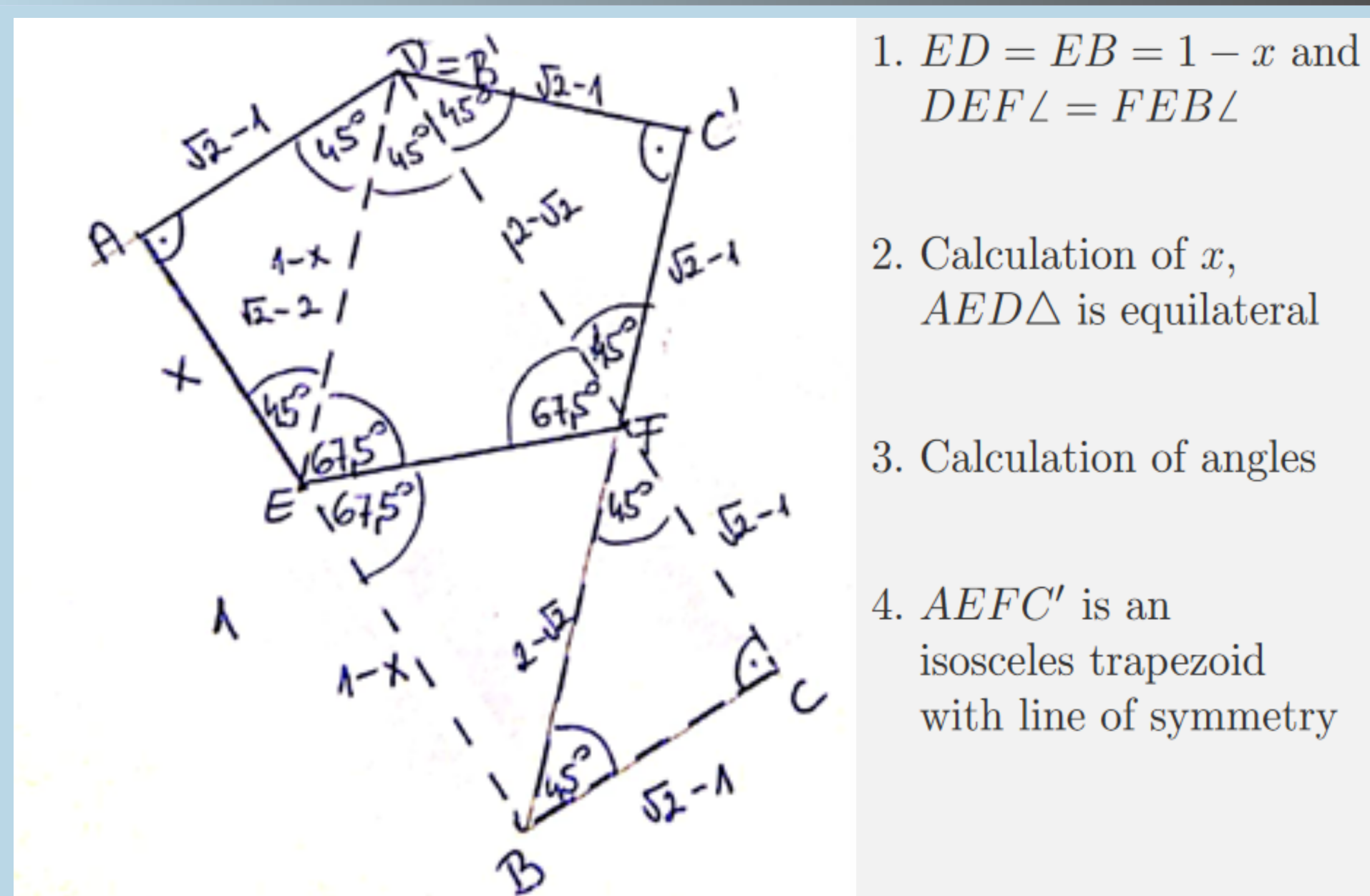
Result

	recognition	proof	non-proof argument	missing argument
symmetry	44	3	12	29
equal sides	39	18	9	12

Question

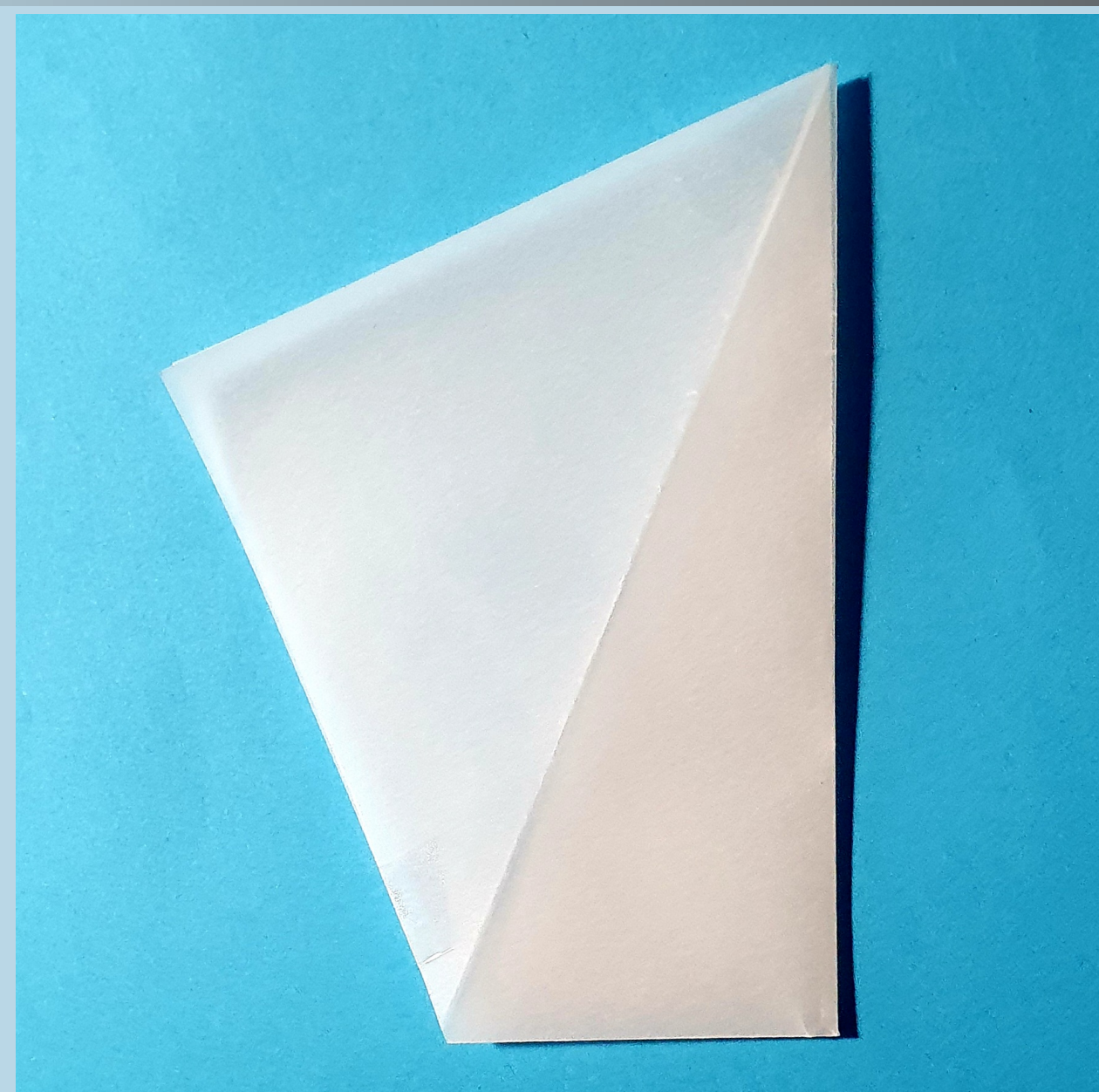
What is the explanation for this striking result?

Proof (symmetry)



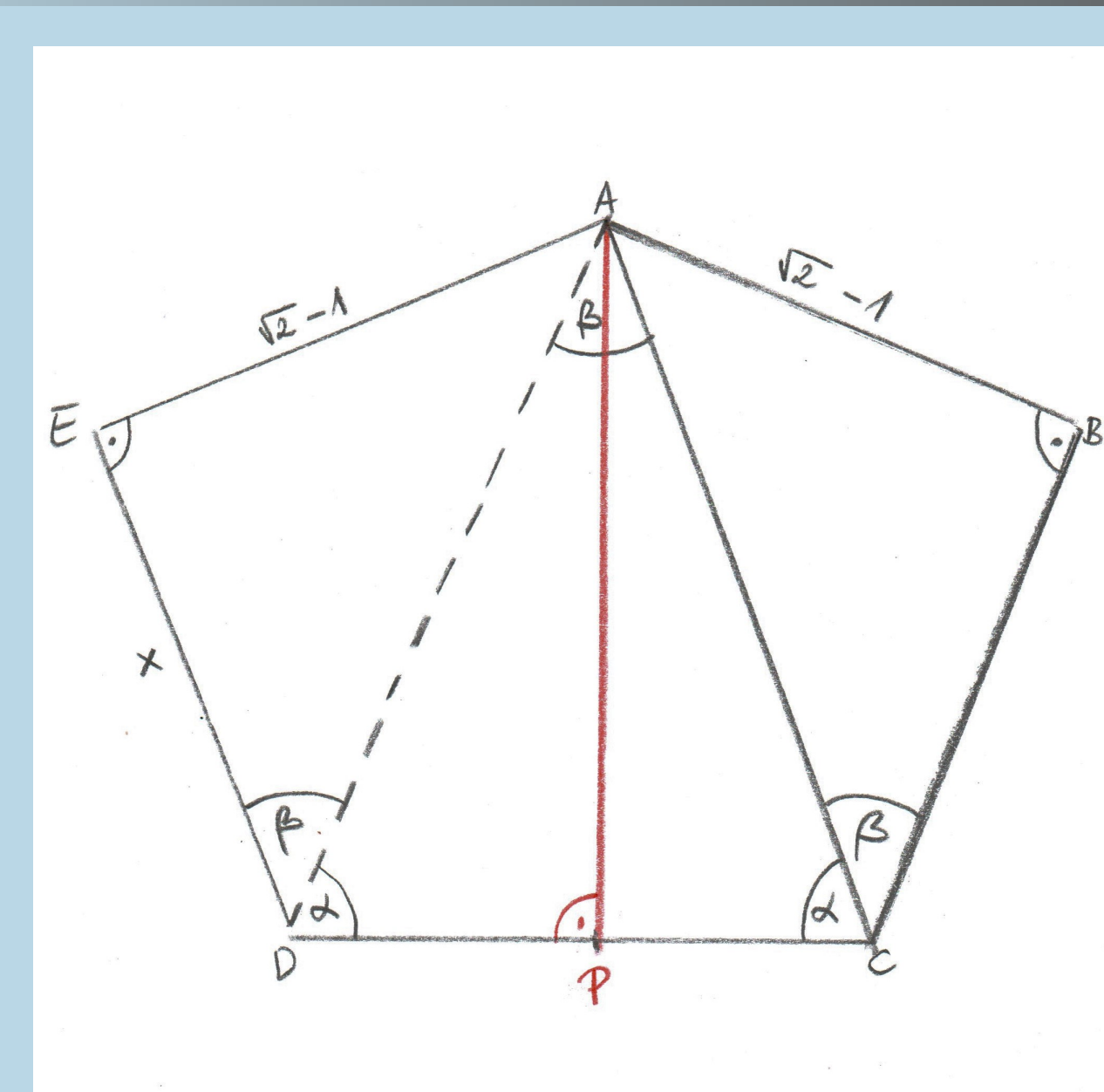
The student proved that the trapezoid $AEFC'$ has a line of symmetry.

Non-proof (symmetry)



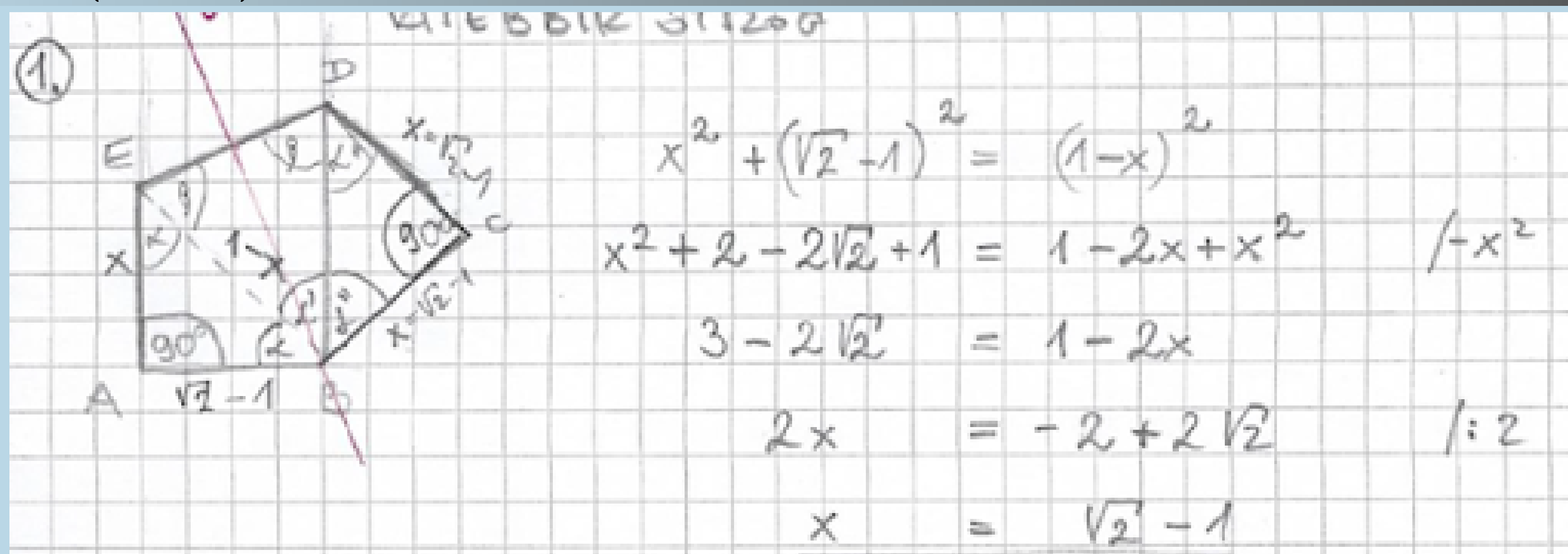
Two parts of the pentagon can be overlapped by folding.

Missing argument (symmetry)



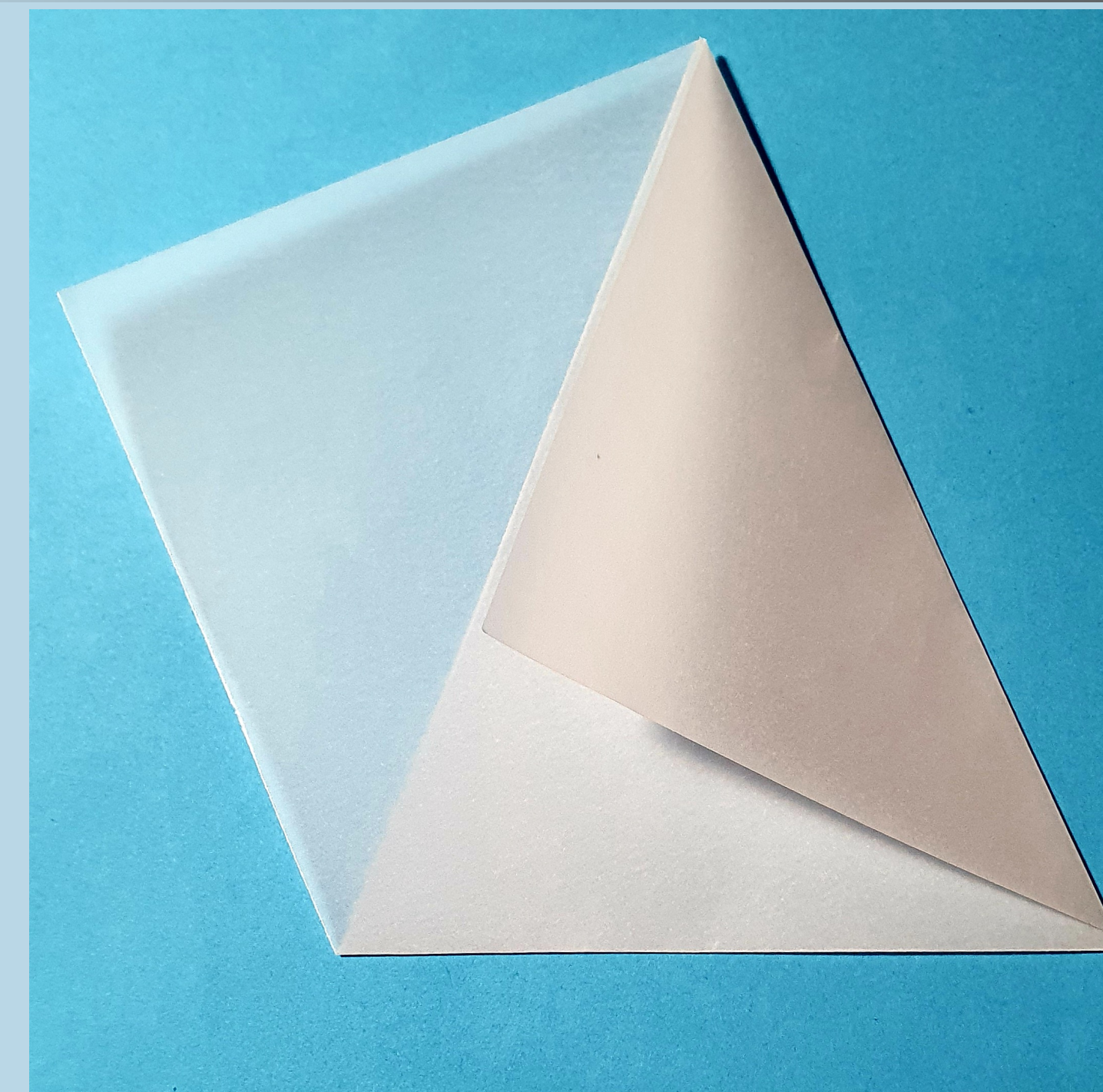
The symmetry axis is drawn correctly, but there is no justification for the symmetry.

Proof (sides)



The property of equal sides has been proved by students using various methods, but the most common argument is the Pythagorean theorem.

Non-proof (sides)



Triangles at right-angled vertices are isosceles triangles.

Discussion

Visual perception and empirical experience affect proving and argumentation. Students had to identify problems in a paper-folded shape on their own. In the case of symmetry, easily detected by perception, as brain research suggests, the need for proof is often neglected, and argumentation is almost entirely limited to an empirical demonstration. In the case of congruent segments, human perception is uncertain, and the conjecture is more likely to be proven by learners. The cognitive uncertainty has triggered students' interest in problem solving and proof. *An open-ended didactic situation that builds on the researcher's interest in students promotes recognition of the proof-needed situation.*

References

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